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# GEOMETRICAL PATTERNS IN THE PRE-CLASSICAL GREEK AREA. PROSPECTING THE BORDERLAND BETWEEN DECORATION, ART, AND STRUCTURAL INQUIRY

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Abstract. — Many general histories of mathematics mention prehistoric "geometric" decorations along with counting and tally-sticks as the earliest beginnings of mathematics, insinuating thus (without making it too explicit) that a direct line of development links such decorations to mathematical geometry. The article confronts this persuasion with a particular historical case: the changing character of geometrical decorations in the later Greek area from the Middle Neolithic through the first millennium BCE.

The development during the "Old European" period (sixth through third millennium BCE, calibrated radiocarbon dates) goes from unsystematic and undiversified beginnings toward great phantasy and variation, and occasional suggestions of combined symmetries, but until the end largely restricted to the visually prominent, and not submitted to formal constraints; the type may be termed "geometrical impressionism".

Since the late sixth millennium, spirals and meanders had been important. In the Cycladic and Minoan orbit these elements develop into seaweed and other soft, living forms. The patterns are vitalized and symmetries dissolve. One might speak of a change from decoration into *art* which, at the same time, is a step *away* from mathematical geometry.

Mycenaean Greece borrows much of the ceramic style of the Minoans; other types of decoration, in contrast, display strong interest precisely in the formal properties of patterns – enough, perhaps, to allow us to speak about an authentically mathematical interest in geometry. In the longer run, this has a certain impact on the style of vase decoration, which becomes more rigid and starts containing non-figurative elements,

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without becoming really formal. At the breakdown of the Mycenaean state system around 1200 BCE, the "mathematical" formalization disappears, and leaves no trace in the decorations of the subsequent Geometric period. These are, instead, further developments of the non-figurative elements and the repetitive style of late Mycenaean vase decorations. Instead of carrying over mathematical exploration from the early Mycenaean to the Classical age, they represent a gradual sliding-back into the visual geometry of earlier ages.

The development of geometrical decoration in the Greek space from the Neolithic through the Iron Age is thus clearly structured when looked at with regard to geometric conceptualizations and ideals. But it is not linear, and no necessity leads from geometrical decoration toward geometrical exploration of formal structures (whether intuitive or provided with proofs). Classical Greek geometry, in particular, appears to have its roots much less directly (if at all) in early geometrical ornamentation than intimated by the general histories.

Résumé. — Motifs géométriques dans l'aire de la Grèce préclassique. Exploration des frontières entre décoration, art et recherche de structures. — Nombre d'histoires générales des mathématiques évoquent aux tout débuts des mathématiques les décorations « géométriques » de la préhistoire, en même temps que l'opération de compter et les baguettes à encoches, suggérant ainsi (sans que ce soit dit explicitement) qu'une ligne de développement directe lie ces décorations à la géométrie en tant que branche des mathématiques. L'article confronte cette conviction à un cas historique particulier: le caractère changeant des décorations géométriques dans ce qui sera l'aire grecque, du néolithique moyen au premier millénaire av. J.-C.

Pendant la période « européenne ancienne » (du sixième au troisième millénaire av. J.-C., dates obtenues à l'aide du carbone 14 et calibrées), le développement va de débuts non systématiques et non diversifiés vers un déploiement d'imagination et de variation, suggérant parfois des symétries combinées, mais ressortissant toujours au visuel sans être soumises à des contraintes formelles; ce type de décoration pourrait être appelé « impressionisme géométrique ».

Depuis la fin du sixième millénaire, les spirales et méandres y occupent une place importante. Dans l'orbite cycladique et minoenne, ces éléments se sont transformés en algues et autres formes souples. De la vie est insufflée dans ces dessins et les symétries se dissolvent. On pourrait parler d'une rupture, la décoration devenant art tout en s'éloignant simultanément de la géométrie.

La céramique de la Grèce mycénienne emprunte beaucoup au style minoen; d'autres types de décoration, en revanche, exhibent un fort penchant pour les propriétés formelles des dessins – suffisamment peut-être pour nous permettre de parler d'un intérêt authentiquement mathématique dans la géométrie. Sur la longue durée, ceci aura un certain impact sur le style des poteries décorées, qui devient plus rigide et commence à inclure des éléments non figuratifs, sans qu'ils soient purement formels. Lors de l'effondrement du système étatique mycénien, vers 1200 av. J.-C., cette formalisation « mathématique » disparaît et ne laisse pas la moindre trace dans les décorations de la période suivante, dite géométrique. Celles-ci résultent, en revanche, d'autres développements, ceux d'éléments non figuratifs et répétitifs présents sur les vases décorées de la période mycénienne tardive. Loin de transférer l'exploration mathématique présente au début de l'époque mycénienne à l'âge classique, elles représentent plutôt un retour progressif vers la géométrie visuelle des périodes antérieures.

Examiné à la lumière des conceptualisations et idéaux géométriques, le développe-

ment de la décoration géométrique dans l'aire culturelle grecque, du néolithique à l'âge de fer, apparaît ainsi clairement structuré. Mais il n'est pas linéaire, il ne mène pas nécessairement d'une décoration à caractère géométrique à l'exploration systématique de structures formelles (qu'elles soient intuitives ou accompagnées de preuves). En particulier, la géométrie grecque classique semble plonger ses racines moins directement que ne le suggèrent les histoires générales, dans les anciennes ornementations géométriques (si toutefois il y en a).

## PRELIMINARY REMARKS

How did mathematics begin? And why did the ancient Greeks develop their particular and unprecedented approach to geometry? Such questions are probably too unspecific to allow any meaningful (not to speak of a simple) answer; even if meaningful answers could be formulated, moreover, sources are hardly available that would allow us to ascertain their validity.

In the likeness of the grand problems of philosophy (Mind-Body, Free Will, and so forth), however, such unanswerable questions may still engage us in reflections that illuminate the framework within which they belong, thereby serving to develop conceptual tools that allow us to derive less unanswerable kindred questions. The pages that follow are meant to do this.

They do so by analyzing a collection of photographs which I made in the National Archaeological Museum and the Oberländer Museum in Athens in 1983, 1992 and 1996, representing geometrical decorations on various artefacts, mostly ceramics; those of them which are essential for the argument are reproduced below.[1] All the artefacts in question were found within, and thus connected to cultures flourishing within, the confines of present-day Greece (Crete excepted). The earliest were produced in the sixth millennium BCE (calibrated radiocarbon date); the youngest belong to the classical age.

General histories of mathematics often identify geometrical patterns along with counting and tally-sticks as the earliest beginnings of the field.[2]

[1] All items are already published and on public display. The photos used here are all mine.

[2] In a sample of eleven works which I looked at, six began in that way: [Smith 1923], [Struik 1948], [Hofmann 1953], [Vogel 1958], [Boyer 1968] and [Wußing 1979]. [Cantor 1907], [Ball 1908] and [Dahan-Dalmedico & Peiffer 1982] take their beginnings with the scribes of the Bronze Age civilization. So does [Kline 1972] on the whole, even though he does discuss pre-scribal mathematics on half a page, and mentions "geometric

Mathematicians (and in this respect historians of mathematics belong to the same tribe) tend to assume that what *we* describe in terms of abstract pattern and shape was also somehow meant by its producers to deal with pattern and shape *per se*, or was at least automatically conducive to interest in these; this is never stated explicitly, but it is an implied tacit presupposition. At least for members of our mathematical tribe it seems a reasonable presupposition.

When first running into the objects rendered in my photographs, I was indeed struck by the easily distinguishable trends in the changing relation of these patterns to geometrical inquiry and thought (what I mean by this beyond "interest in pattern and shape *per se*" will be made more explicit in the following); I also noticed, however, that development over time could as easily lead away from mathematical geometry as closer to it. Mathematics is no necessary, not even an obvious consequence of the interest in visual regularity (which, on its part, appears to be rather universal). Not every culture aims at the same type of regularity, and the interest in precisely mathematical regularity is a choice, one possible choice among several.

On the other hand, the universal human interest in regularity – that "sense of order" of which Gombrich [1984] speaks – *may* certainly lead to systematic probing of formal properties of symmetry, similarity, etc. Whether such inquiry is connected to some kind of proof or argument or not (which mostly we cannot know), there is no reason to deny it the label of "mathematics" (or, if we prefer this distinction and that use of the term, "ethnomathematics", as an element of mathematical thought integrated in an oral or pre-state culture). In order to distinguish these cases from such uses of patterns and shapes whose intention and perspective we are unlikely to grasp through a characterization as "mathematics", we need to develop concepts that reach further than the conventional wisdom (or, with Bacon, "idols") of our tribe.

My purpose is thus primarily a clarification of concepts which may permit us to look deeper into the relation between decorative patterns and mathematics; it is neither the history of artistic styles nor the links

decoration of pottery, [and] patterns woven into cloth" in these eight words. Chapter 1 on "Numeral Systems" of [Eves 1969] contains half a page of speculations on "primitive counting".

between cultures. For this reason I do little to point out the evident connections between, for example, the decorations found on Greek soil and the styles of the Vinča and other related Balkan cultures.

The gauge is deliberately anachronistic, and I make no attempt to interpret the artefacts which I discuss in their own practical or cultural context (although I do refer occasionally to their belonging within a specific framework – deliberate anachronism should never be blind to *being* anachronism). My purpose is, indeed, not to understand this context but to obtain a better understanding of the implications of that other blatant anachronism which consists in reading early decorations in the future-perfect of mathematics – an anachronism which can only (and should only, if at all) be defended as a way to understand better the nature *of mathematics* and the conditions for its emergence.

Though this was not on my mind when undertaking the investigation, my approach can be described as a *hermeneutics of non-verbal expression* – "hermeneutics" being so far taken in Gadamer's sense that the expression of "the other" is *a priori* assumed, if not to be "true" (obviously, expressions that do not consist of statements possess no truth value) then at least to be "true to an intention". Whereas the habitual ascription of a "mathematical intention" to every pattern and symmetry can be compared with that reading of a foreign text which locates it straightaway within the "horizon" of the reader, my intention here may be likened to Gadamer's *Horizontverschmelzung*, "amalgamation of horizons". In agreement with Gadamer's notion of the hermeneutic circle I presuppose that such an amalgamation is possible, that our present horizon can be widened so as to encompass that of the past "dialogue partners" (yet without sharing Gadamer's teleological conviction that this amalgamated horizon can also be said to be the *true* implied horizon of the partners; the wider horizon remains ours, and remains anachronistic).[3] As we shall see, this requires that our wider horizon transcends that of the mathematical tribe.

As affirmed emphatically by Gadamer, hermeneutics is no method, no prescription of the steps that should be taken in the interpretation of a

[3] See [Gadamer 1972, pp. 289f and *passim*]. The stance that the amalgamated horizon is the true implied horizon of the partners corresponds to that kind of historiography of mathematics according to which contemporary mathematicians, those who have insight into the tradition as it has unfolded, are the only ones that are able to understand the ancient mathematicians and thus those who should write the history of mathematics.

foreign text. This is certainly no less true for a "hermeneutics of non-verbal expression". For this reason, the conceptual tools that emerge during the investigation – in particular a notion of "geometrical impressionism", and a particular (tentative) distinction between "art" and "decoration" – cannot be adequately explained in abstraction from the material and developments they serve to elucidate, however much they may afterwards reach beyond this particular material and these particular developments.

One key concept, however, must be confronted before we can begin the discussion of whether any development points toward mathematical geometry or not: that of "mathematics". Chronologically, mathematics may be said to begin at any point in time at least since the moment when the first mammals integrated sense impressions as representations of a permanent object, thereby bringing forth that *unity* which, according to ancient and medieval metamathematics, is the "root of number". Evidently, no meaningful precise cut in this continuum can be established; but I shall use as a heuristic delimitation the principle that mathematics presupposes coordination or exploration of formal relations, based on at least intuitively grasped understanding of these. Since my concern is whether developments lead "toward mathematics" or away from it rather than the decision whether a particular pattern *is* mathematics, the inescapable imprecision of the delimitation will be no severe trouble.

As far as the other aspect of the investigation is concerned – the roots of the particular Greek approach to geometry – no conceptual innovation is needed. The results – first of all that nothing in the "geometric" style of the ninth through seventh century BCE points toward the emergence of "rational geometry" – will emerge through the analysis.

Since the purpose of the investigation is the sharpening of conceptual tools (and, to a lesser extent, analysis of the historical process *within* the Greek area), I shall permit myself to date the items I discuss as done in recent years by the museums and in the catalogue of the Archaeological Museum [Petrakos 1981],[4] relative chronology being all I need. As far as the second millennium BCE and the late third millennium are concerned, the dates seem to be derived from Egyptian and Near Eastern historical chronology, and thus to be *grosso modo* correct. Earlier dates (presented

[4] In 1983, the displayed dates for the older period in the Archaeological Museum (not yet coordinated with the catalogue) were even younger than now.

in [Petrakos 1981, p. 11] as "generally accepted conclusions") appear to be uncalibrated radiocarbon dates, since they coincide with what other publications (*e.g.* [Gimbutas 1974]) give as uncalibrated dates for the same sites and periods; when asked, Dr. M. Vlassopoulou of the Museum confirmed my hunch.[5] Approximate translation into true historical date (as determined by dendrochronology) can be made by means of this table, based on [Watkins (ed.) 1975, pp. 118–124] and [Ferguson *et al.*, p. 1976]:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Uncalibr. radiocarbon date | 2000 | 2500 | 3000 | 3500 | 4000 | 4500 | 5000 | 5500 |
| Approx. historical date | 2500 | 3240 | 3720 | 4410 | 4880 | 5400 | 5900 | 6450 |

All dates are BCE; the translations of radiocarbon date into historical date are with a margin of ±50 to 100 years, to which comes the imprecision of the radiocarbon dating itself.

Even in the naming of periods I follow the Museum Catalogue. This implies that what is here spoken of as the "Late Neolithic" will be spoken of as the "Chalcolithic" in the majority of recent publications.

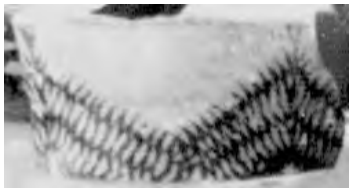
In all figure captions, ArchM stands for the National Archaeological Museum in Athens. ObM stands for the Oberländer Museum, Kerameikos, Athens. All dates are evidently BCE.

## 'OLD EUROPE'

The sequence #1 to #24 represents – at the level of generalization on which I move here – a fairly uniform development that passes through several stages but is never radically interrupted. Chronologically it spans the period from the early fifth through the late third millennium (uncalibrated radiocarbon dates). Since the third millennium items all belong to the Cycladic area, where the influence from the "Kurgan" intrusion and interruption of the more northerly branches of the Balkan culture was only weakly felt, the whole sequence must be connected to the culture of

[5] Dr. Vlassopoulou also procured for me the date and origin of artefacts which were displayed in the Museum without any such indications and corrected dates that had been wrongly indicated in the exhibition. I use the opportunity to express to her my sincere gratitude.

"Old Europe" and its Cycladic offspring.[6] Restriction to the Greek area has the added advantage that we avoid whatever particular effects may have been caused by the rise of large, more or less town-like settlements in the Vinča culture – *cf.* [Gimbutas 1974, p. 22].



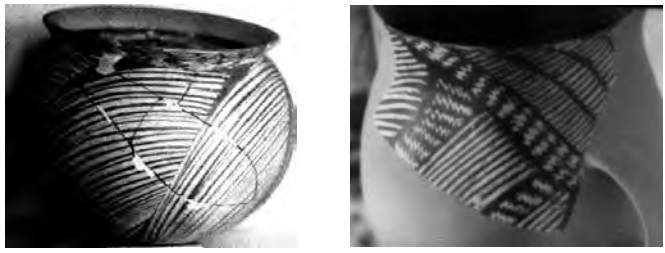
*Photograph # 1 (left). ArchM, Museum N°5918. Middle Neolithic, 'Sesklo style', 5th millennium.*
*Photograph # 2 (right). ArchM, Museum N°5919. Middle Neolithic, 'Sesklo style', 5th millennium.*

*Photograph # 3. ArchM, Museum N°51918. Middle Neolithic, 'Sesklo style', 5th millennium.*

Several sub-periods can be distinguished. Photographs #1-3 are representative of the Middle Neolithic Sesklo period. All items reflect interest in bands of acute angles, triangular organization and concentric rhombs (the latter in #2 and in other items not shown here). Only straight lines are made use of, and no attempt is made at integrating the order that

[6] See [Gimbutas 1973a, 1973b]. The more disputed aspects of Marija Gimbutas' description of the cultural sequence are immaterial in the present connection – thus whether her "Kurgan" pastoralists are identical with the Proto-Indo-Europeans (*cf.* [Mallory 1989, pp. 233–243 and *passim*]).

characterizes the single levels into a total system, nor to correlate the pattern with the geometry of the object which is decorated – if we analyze #1 we first see a macro-level where three zigzag-lines run in parallel. The lower of these, however, goes beyond the inferior edge of the vase. Each segment of the zigzag line is in itself a zigzag-line, but made according to principles which differ from those of the macro-level; we may characterize it as a band of spines. These segments, furthermore, meet in a way which lets their spines cross each other. Each segment is clearly thought of in isolation.



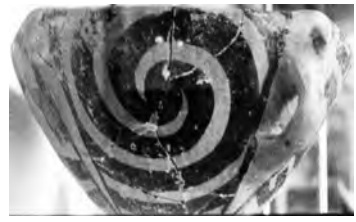
*Photograph # 4 (left). ArchM, Museum N°8051. Lianokladi, Middle Neolithic, 'scraped ware', 5th millennium.*
*Photograph # 5 (right). ArchM, Museum N°8066. Lianokladi, Middle Neolithic, 'scraped ware', 5th millennium.*

The beautiful, more or less contemporary "scraped ware" from Lianokladi is even less formal in its use of "geometric" decoration (see #4–5); both on the level of global organization of the surface and regarding the internal organization of each segment, irregularity is deliberately pursued. The fragments from the pre-Dimini-phase (#6) of the Late Neolithic ("Chalcolithic" would be better, copper being in widespread use in the Old European culture during this period) exhibit some more variation than the Sesklo specimens (spirals turn up), but convey the same overall impression.

The decorations belonging to the Dimini phase of the Late Neolithic (#7–13) are somewhat different. Now larger parts of the surfaces are covered by geometrically coherent decoration, but in most cases still only *parts* of even larger surfaces. In #7, a chessboard pattern is partly covered in two places by a series of parallel lines – lines which, furthermore, run in a direction which deviates slightly but unmistakably and deliberately

*Photograph # 6. ArchM, various museum numbers. Fragments, Late Neolithic, pre-Diminian phase, 4300–3800.*

from the closest axis of the chessboard. The outer edge of the chessboard is also wholly incongruous with both the sides and the diagonal of the pattern itself, even though inclusion of part of the blank space to the left would have permitted agreement with the diagonal. In #8 a spiral-system is clipped in a way which demonstrates that it is imagined as cut out from a larger spiral. No attempt is made to unite the spiral with the geometrical conditions offered by the vase – as in an amateur photo, the motif is one thing and the frame is another. Only #9 suggests that the conditions arising from the surface to be decorated and the geometry of the decorating motif are thought of as *one* problem.



*Photograph # 7 (left). ArchM, Museum N°5925. Late Neolithic, 'Dimini style', 4th millennium.*
*Photograph # 8 (right). ArchM, Museum N°5932. Late Neolithic, 'Dimini style', 4th millennium.*

The decorations of the Middle Neolithic were constructed from straight lines, we remember. The use of curved lines, especially in the form of

*Photograph # 9 (left). ArchM, Museum N°5934. Late Neolithic, 4th millennium.*
*Photograph # 10 (right). ArchM, Museum N°592. Late Neolithic, Dimini, 4th millennium.*

spirals, is thus an innovation. The combination of two spirals in #9 – each of roughly the same type as in #8 – presents us with another kind of innovation: it can be understood as a geometrical restructuration of less complex material. It allows coherent decoration of the irregular surface, but only at the cost of eclecticism. The core of the lower spiral has been turned 90° with respect to its counterpart, allowing it thus to be flattened and broadened. Furthermore, regions where too much space is left uncovered by the meander are filled out by triangles. The purpose of the pattern is decorative, rather than geometrical exploration.[7]

Other items confirm a tendency toward greater variation in comparison with those of the Middle Neolithic, but the patterns always remain decorative and eclectic, and nothing suggests that geometrical regularity is pursued for its own sake. In #10, the spaces left open by the bands of parallel lines are filled out by figures of highly heterogeneous character (spiral, circular segments, zigzag-lines in pointed pseudo-ellipses). In the bands of parallel lines, the number of lines varies from one band

[7] Evidently, this dichotomy does not exhaust what can be said about a geometrical pattern. For instance, patterns may possess symbolic functions; but even if we follow Marija Gimbutas and interpret the meanders as snake symbolism, we have to observe that meanders meant as formalized snakes may be used in a way that suggests geometrical exploration, or they may be located eclectically, as suggested by decorative intuition.

to another, and the suggested mirror symmetry between the left and right pseudo-ellipses is contradicted by the translational symmetry of the zigzag-lines which they enclose (which, given the general eclecticism of the composition, is not likely to represent a deliberate experiment with symmetry breaking). Geometrical regularity at the visual level furnishes the material, but the governing principles and the overriding concerns are different: leaving aside their further meaning for the artist we may say that aesthetic sensibility is more important than regularity even at the visual level. Moreover, symmetry appears to be disregarded when inconvenient, not first deliberately suggested and then consciously broken as (for example) in certain Persian carpets, which would still be a kind of sophisticated geometrical exploration.

*Photograph # 11. ArchM, Museum N°5931.*
*Late Neolithic, 'Dimini style', 4th millennium.*

In #11 we find the same eclecticism as in #7 (it is only one of several specimens in the Museum that follow the same fundamental model): on the back, another chessboard pattern is partially covered by a band of parallel lines, even this time slightly slanted (see the photo in [Matz 1962, p. 28]); between the two chessboards, both #7 and #11 carry a two-dimensional in-law of the rectangular meander (with similar hatchings in both specimens). The geometrical eclecticism of the decoration is thus hardly a random phenomenon, it must be assumed to be governed by deliberate considerations that are external to the pattern itself – perhaps a symbolic interpretation given to the constituents and to their mutual relation.

All the more striking is the geometrical carelessness demonstrated in the upper left corner of the chessboard of #11, where fields that should have been black have become white, and vice versa. Similar seeming

*Photograph # 12. ArchM, Museum N°5937.*
*Late Neolithic, Sesklo, 4th millennium.*

carelessness is found in other cases (*cf.* the fragments in #13), and hence better understood as emphasis on visual impression and absence of formal mathematical constraint.

Comparing the Late Neolithic decorations with those of the Middle Neolithic we may conclude that greater geometrical phantasy and sensitivity makes itself felt. None the less, the visual effect remains the overriding concern, and the over-all impression which results from application of a geometrico-mathematical gauge is one of unworried eclecticism. Formal constraints – be they based on counting or on rotational, translational or mirror symmetry – are relatively unimportant as soon as they go beyond what is visually obvious for the geometrically innocent mind. At the level of the visually obvious, on the other hand, they *are* important: the chessboard pattern is *almost* there even in #11, and the pattern in the left part

*Photograph # 13. ArchM, various museum numbers. Late Neolithic, 'Dimini style', 4th millennium.*

of the same photo exhibits vertical and horizontal translational symmetry as well as symmetry against a rotation of 180° (disregarding rather strong metric distortions).

Decorative painting remains abstract, no figurative elements are involved even though figurative sculpture is well represented in the record – #12 shows a Late Neolithic piece from Sesklo which is itself covered by an abstract pattern (supposed by Gimbutas [1974, p. 144] to be possibly a snake symbolism, but even then a thorough abstraction[8]).

In the photos from the third-millennium Cycladic Early Bronze Age culture (#14–24) certain deviations from this pattern become visible, but no fundamental changes can be traced. Figurative sculpture is still found, at times decorated with abstract patterns (thus #14). Decorations themselves may now involve figurative elements. In #15, four fish and a sun enter an otherwise geometric composition, participating in its highly symmetric design; in #16 (and in many similar "frying pans"), a picture of a boat is surrounded by a geometric pattern,[9] while the whole scene

[8] More thorough indeed than the abstraction of certain Cucuteni zigzag-lines provided with a snake's head [Dumitrescu 1968, pl. 42, 48].

[9] That this pattern is thus likely to represent stylized water is immaterial for the actual

*Photograph # 14. ArchM, Museum N°5698.*
*Melos, Phylakopi I, 2300–2000.*

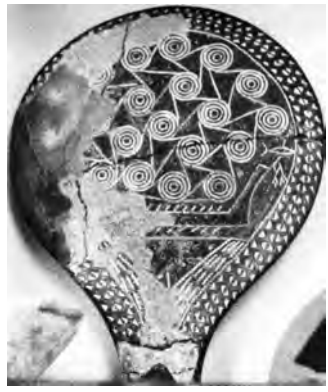
*Photograph # 15. ArchM, Museum N°6140A.*
*Naxos, Early Cycladic II, 2800–2300.*

stands on a female pubic triangle (drawn exactly as on female figurines). Interwoven spirals become a major theme in the decorations,[10] as can be seen in #17 (where the pattern is fully systematic) and in #15 (where close inspection of the lines reveals a lack of regularity). The importance of this complex pattern is perhaps most clearly seen in #16 and #18 (shown as representatives of a large class of similarly decorated pieces), both of which present us not with spirals but with an easy counterfeit: systems of concentric circles (so uniform that they are likely to have been

discussion, since it is anyhow geometrical.

[10] Once more, we need not pursue the possible inspiration from cultures with which the Cycladeans may have been in contact. Interconnected spirals were also popular in the Megalithic Culture(s) to the West, from Malta to Ireland. The critical question is on which level of geometry the motif was used within the Cycladic culture. The Megalithic monuments themselves vary in this respect, from strict organization – a specimen from Tarxien, Malta, is reproduced in [Guilaine 1981, p. 970] – to arrangements even more loose than #16 – a specimen from Newgrange, Ireland, is in [Mohen 1984, p. 1536].

made by means of a multiple compass) connected by straight or slightly curved lines. Only one item, however, can be said with some justification to explore the possibilities of a geometrical pattern formally, *viz* #17, an engraved steatite box. In #15, on the contrary, the real symmetry is less than the one suggested by immediate inspection (see the centres of the spirals).



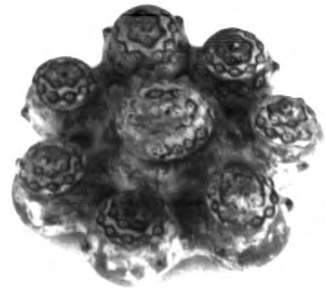
*Photograph # 16. ArchM, Museum N°5053. Syros, Early Cycladic II, 2800–2300.*

*Photograph # 17. ArchM, Museum N°5358. Naxos, Early Cycladic II, 2800–2300.*

Other items exhibit in stronger form this contrast between seeming regularity at the level of immediate visual impression and random irregularity below this level – henceforth I shall speak of "geometrical impressionism". In #19 the apices of the black triangles of one band are sometimes adjusted to the band above, sometimes they move without system with respect to the bases of the triangles in this band; the number of strokes in the hatchings varies – in some triangles they run parallel to the left edge (at least ideally speaking), in others they cut it obliquely, in still others they are

*Photograph # 18 (left). ArchM, Museum N°6180. Syros, Early Cycladic II, 2800–2300.*
*Photograph # 19 (right). ArchM, Museum N°5358. Naxos, Early Cycladic II, 2800–2300.*

vertical. In the similar though cruder pattern of #20, the hatching is mostly parallel to the right edge but occasionally (and without system) to the left edge. In #16, the single systems of concentric circles have been located as best they could, in order to fill out the space left open between the border and the boat; in the interior part of the pattern, moreover, most of the systems are connected to six but some to five or seven other systems: no idea is obviously present that systems of circles "should" be arranged with hexagonal symmetry – *cf.* also #21, where a central "circle" is surrounded by seven, not six other "circles".



*Photograph # 20 (left). ArchM, Museum N°5171. Taphos, Early Cycladic II, 2800–2300.*
*Photograph # 21 (right). ArchM, Museum N°6185. Early Cycladic II, 2800–2300.*

*Photograph # 22 (left). ArchM, Museum N°5153. Syros, Early Cycladic II, 2800–2300.*
*Photograph # 23 (right). ArchM, Museum N°8874. Raphina, early Helladic, 3d millennium.*

Similarly, the suggested star of the "frying-pan" of #22 is, at most, *a suggestion* of stellar symmetry. In #23, the number of hatching lines in the cross-hatched triangles is sometimes 8 and sometimes 9. Even #24, apparently fully symmetric (apart from some topological distortion), turns out not to be so when we start counting the dots.

Seen as a whole, the development of geometric patterns in the Old European period is thus one from unsystematic and rather undiversified beginnings in Middle Neolithic Sesklo toward great phantasy and variation and even sophisticated combined symmetries in the third millennium, but throughout the period largely restricted to the visually obvious, and – with at most a single exception in the material shown above (*viz* #17) – never formally carried through: geometric structure is and remains subservient to other purposes, where we are unable to extricate aesthetics or decoration from symbolization.[11]

During the whole span of the Neolithic and the third millennium, decorative *painting* remains almost exclusively abstract and non-figurative, even though figurative *sculpture* is known from all periods. Only the "frying pans" of the Cycladic third millennium contain figurative elements, at times (#15) integrated in the geometric symmetry of the design, at times

[11] Evidently, questions presupposing that these occur in additive and thus separable combination are probably misguided – who would ever claim that a *Pietà* carries less religious feeling or meaning because the painting has a strong aesthetic impact?

*Photograph # 24. ArchM, Museum N°5225. Early Cycladic II, 2800–2300.*

(#16) an independent condition to which the geometric structure has to submit.

## IN MINOAN ORBIT

The Middle Cycladic culture of the first half of the second millennium was a continuation of earlier Cycladic cultures [Christopoulos & Bastias (eds) 1974, p. 140], and distinct from the culture of Minoan Crete. From the point of view of the present investigation, however, the *transformation* of earlier practices as well as the Minoan affinities soon become evident.

*Photograph # 25 (left). ArchM, Museum N°5857. Orchomenos, Middle Helladic, 2000–1500.*
*Photograph # 26 (right). ArchM, Museum N°5876. Orchomenos, Middle Helladic, 2000–1500.*

The period is presented by photos #25–36. #25–26 are from pre-Mycenaean Orchomenos in Boeotia; they are included in contrast to the following and as supplementary examples of impressionistic geometry. In #25, the number of strokes in each band is almost constant-but not quite; moreover, the two bands of 11 strokes (against the normal 12) could easily have contained 12 strokes – the bands are apparently made from above, and in the lower end the distance between the strokes is augmenting in these two bands. In #26, nothing is done in order to harmonize the apices of the two sets of adjacent zigzag-lines.

*Photograph # 27 (left). ArchM, Museum N°5286. Melos, Phylakopi, between I and II. C. 2000.*
*Photograph # 28 (right). ArchM, Museum N°5759. Melos, Phylakopi II, early 2nd millennium.*

#27 – the first Cycladic specimen – differs from the Orchomenos samples by a free experimentation with cross-hatching of squares and ribbons that is unconstrained by attempts to achieve symmetry even at the immediate visual level (the hidden side contains the same elements as the one that is shown, but in a very different arrangement); the global arrangement is as eclectic as it is pleasant to a contemporary eye.

The first striking characteristic of the following Cycladic decorations is the presence of figurative painting. This seems alien to the "native tradition" of the area as we know it from the preceding section, and could be interpreted as an indication of cultural diffusion. Diffusion may,

indeed, be part of the "efficient cause" of the change which took place.[12] But diffusion is, as always, a rather empty word which hides at least as much as it explains. In the present case it tends to veil the problem why figurativeness was learned from the Egyptians (if we suppose Egypt to be the source – other sources would raise similar problems) while the "canonical system" of Egyptian figurative art[13] was certainly not. The diffusionist explanation, furthermore, leaves aside the question how the diffused cultural element became part of an integrated cultural system: which specific character was "figurativeness" to acquire in the Cycladic context?[14]

A characteristic example of this specific character is found in #28. Spirals are essential, as in so many decorations since the fourth millennium – but the spirals are undergoing a process of dissolution, they have become aquatic plants growing out of the sea floor. *Figurative painting does not come in as a complement or substitute* (as it was to do in the Late Geometric period); *instead, the character of the traditional decorative pattern* (already giving up the quest for symmetry even at the visual level in #27) *is transformed and becomes figurative itself.*

The same feature can be observed in #29. The spirals are more geometrical, but they are growing out of a common floor, and they are deliberately differentiated (we notice that they are seven in number, *cf.* #21). The edging that surrounds them, moreover, is no repeated

[12] I leave aside the question why Paleolithic cultures tend to have figurative drawing and painting, whereas Neolithic ceramics is almost always abstract and "geometric", and "civilizations" reintroduce the figurative element. Since Paleolithic and Mesolithic cultures as a rule *have* no ceramics but may use abstract decorations on other surfaces (examples in [Otto 1976, pp. 45–49]), and since Neolithic societies may produce figurative sculpture, the real issue is more complex than the oft-repeated three-step scheme might make us believe. That part of the answer which goes beyond the fate of materials (ceramics survives, wooden tools rarely, tattooings almost never) may have to do with the social division of labour.

In any case, the development of civilization in the Cycladic area is in itself correlated with cultural contact and learning. Diffusion of artistic styles thus cannot be separated from the effects of the civilizing social process.

[13] See [Iversen 1975]. Central elements of the canonical system are the use of the square grid and the observance of strict proportions between the single parts of the human (or animal) body – elements which together contribute strongly to the formal character of Egyptian art and which sets it decisively apart from anything Cycladic and Minoan.

[14] These points are commonplace objections to diffusionism. They are repeated because they arise specifically in the present context.

*Photograph # 29 (left). ArchM, Museum N°5740. Melos, Phylakopi II, early 2nd millennium.*
*Photograph # 30 (right). ArchM, Museum N°5804. Melos, Phylakopi II, early 2nd millennium.*

abstract shape but a band of not too similar leaves.

Other items are not as easily identified as missing links between geometry and vegetation. The aquatic plants of #30 are not derived from a geometric figure. Yet if we compare #30 with for instance #18 and #20 we shall still encounter evidence for a transformation of the geometrical principles. The latter two are repetitive, in principle they exhibit rotational symmetry. The decoration of #30 is also constructed from a repetitive basis, but now the symmetry is intentionally broken – not, as in the preceding period, in a way which can be characterized as a secondary deviation from a suggested principle, but in a way that cannot avoid being noticed and which must have been meant to be part of the immediate impression.

The upper part of the decoration is non-figurative, consisting of connected systems of concentric circles. Whereas the concentric circles of #16 and #18 were drawn with a multiple compass (which is to return in later periods), those of #30 are not drawn with precision; nor are they, for that matter, always complete circles (*cf.* also #31–32). They could be described as *living* patterns.

A similar conclusion can be drawn if we compare #33 with #27. In both cases cross-hatched ribbons are seen; but whereas the spaces which are left open between the ribbons are filled out with squares in the early

*Photograph # 31 (left). ArchM, Museum N°5758. Melos, Phylakopi II, early 2nd millennium.*
*Photograph # 32 (right). ArchM, Thera collection N°58. Thera, 1550–1500, Minoan import.*

*Photograph # 33 (left). ArchM, Museum N°5757. Melos, Phylakopi III, 1600–1400. Local pottery with Minoan influence.*
*Photograph # 34 (right). ArchM, Museum N°5803. Melos, Phylakopi III, 1600–1400. Probably Minoan import.*

Middle Cycladic piece, plants are used to hold the *horror vacui* aloof in #33. We may also observe that one of the ribbons of #33 is twined, while those of #27 are not.

For comparison with later developments, finally, the paintings of #34 (the freely swimming octopus) and #35 (stylized ivy etc.) should be taken note of.

Most of the rare figurative motifs of the earlier period were artefacts (boats) or, if living beings, made as stiff as artefacts (the fish of #15)

*Photograph # 35 (left). ArchM, Museum N°5789. Melos, Phylakopi III, 1600–1400. Probably Minoan import.*
*Photograph # 36 (right). ArchM, Museum N°1838. Thera, 16th c.*

(the female pubic triangle of #16 and other "frying pans" seems to be symbolic rather than really figurative). Most Middle Cycladic and Minoan figurative motifs, on the other hand, are plants (and among these, softly waving aquatic and twining plants dominate); animals do occur, but the octopus of #34 as well as the duck of #36 are drawn with soft, almost vegetative lines. Human artefacts with their sharply cut contours are avoided in ceramic decorations (though not in the Thera frescoes).

In the perspective of the present study, the basic characteristic of the decorations of this period is thus the transformation of geometrical patterns and motifs: the patterns are vitalized, they are re-conceptualized as living creatures or quasi-living, moving lines.[15] Symmetry is upheld as an underlying idea but only to be deliberately broken, becoming the symmetry of a garden rather than that of fully planned human creation.[16] In spite of the inherent danger of the anachronism, one is tempted to describe the development of the geometrical pattern as one

[15] The development is thus a reversal of that stylization of snakes into abstract lines which Gimbutas suspects in #12, and which is unmistakable in certain Cucuteni decorations (*cf.* note 8).

[16] An editor objects to this metaphor that "un jardin de Le Nôtre" seems to belong to the category of the fully planned. Actually, this exception to what gardens are in most human cultures illustrates the point: Le Nôtre's gardens came out of his studies of perspective theory and architecture, and they try to avoid the spontaneity of trees planted symmetrically but growing in asymmetric ways – or at least to reduce asymmetry to the level where it goes unnoticed, as in pre-Cycladic "impressionistic geometry".

from *decoration* into *art.*[17]

## THE TWO FACES OF MYCENAEAN GREECE

The photos from Mycenaean Greece are ordered in two separate sequences, #37–56 ("sequence I") and #57–70 ("sequence II"). Sequence I represents decoration of non-ceramic artefacts; sequence II shows what happened to pottery decoration. Whereas the latter sequence derives originally from Minoan, Cycladic and closely related styles, and therefore shows the gradual transformation of a borrowed aesthetics, the former is, since its beginnings, completely different from these (and no less different from the Orchomenos decorations #25–30). It may hence legitimately be regarded as an expression of a "native" style of the Mycenaeo-Greek tribes.

#37–55 come from the "Grave Circle A" excavated by Schliemann (but similar artefacts have been found in other Mycenaean contexts, for example in 15th-c. Aïdonia). Except for the stone stela of #37, all of them must be characterized as *Kleinkunst.*

The geometry of this sequence differs in character, not only from the Middle Cycladic and Minoan but also from the Old European style, concerned as the latter had been with visual impression, often geometrically regular at the level of immediate perception but imprecise below that.

The gold roundel of #38 may serve to highlight this difference. The circular edge is made by means of a compass (by leaving the central

[17] This distinction could be thought of in terms of the classical dichotomy where art – *poiesis* – was expected to be characterized by some kind of *mimesis* whereas decorative friezes were not; in this sense, the development in question is evidently but trivially pointing toward art. Less trivial and more pertinent in Kandinsky's century would be the observation that Cycladic decoration, as we move from the third into the second millennium, becomes increasingly bold when dealing with the tension between regularity and irregularity – in the end assigning to regularity the role of a decisive but hidden governing principle. (Better perhaps, hidden but decisive, *viz* if anything superficially mimetic shall be more than a heap of haphazard ingredients – the "classical" and the contemporary view of art are certainly more intimately connected than a naive reading of the above formulations reveals).

I am grateful to my former colleague Paisley Livingston for forcing me to give the reasons for what started as a too facile intuition. I use the opportunity to thank him also for linguistic control. (Already because the text he read was a preliminary version, he is obviously not responsible for whatever clumsy phrases I may have produced later in the process).

*Photograph # 37 (left). ArchM, Museum N°1428. Mycenae, grave circle A, shaft grave 5, 1580–1550.*
*Photograph # 38 (right). ArchM, Museum N°20. Mycenae, grave circle A, shaft grave 3, 1550–1500.*

point unerased, the artist has taken care that we do not overlook this fact); so are the circular arcs drawn inside it. The pattern is the one which arises when you try to structure the whole plane homogeneously by means of a compass with constant opening (as done in tree planting *in quincunx*, as it was called in Roman antiquity), or when you draw longer arcs than needed during the construction of a regular hexagon. Even if not concerned with any kind of scientific geometry, the pattern of the roundel is more mathematical in its geometry than anything we have seen thus far, representing a systematic exploration of the properties of the circle.[18] Exact measurement also shows that the six small circles are centred precisely with respect to the equilateral triangles inside which they are drawn.

The roundel in question comes from one of the later graves. If we compare it with the roundel of #40, or the gold belt of #41 (both of which are about one generation older), we find the same hexagonal symmetry

[18] The use of the same roundel type for the scales of the balance in #39 reminds us once more that this geometrical investigation is coupled inextricably with other concerns: that the device is purely symbolic is obvious – is is made from gold foil and thus not fit for carrying the slightest weight; its presence in a grave suggests some kind of religious meaning. This observation, however, does not preclude analysis of its geometrical character.

and, in #40, almost the same circular arcs. But the arcs are not compass-drawn, and they are not drawn through the centre – any attempt to do so would indeed reveal their imperfection. These early specimens already demonstrate a search for the mathematical symmetry of #38, but the final result has not yet been achieved.[19]

*Photograph # 39. ArchM, Museum N°81. Mycenae, grave circle A, shaft grave 3, 1550–1500. A similar balance carries the bee of # 54. Photograph # 40. ArchM, Museum N°669. Mycenae, grave circle A, shaft grave 5, 1580–1550.*

Other pieces from the earliest shaft graves demonstrate a similar interest in geometrical perfection, yet not always as firmly based on mathematical regularity as #38. The square grid of connected spirals in the upper part of #37 is as regular as that of the Early Cycladic steatite box of #17 (exceptional, we remember, in its own time). Its 90° rotational symmetry, however, is determined from the rectangular frame which surrounds the grid and has nothing to do with the intrinsic geometrical properties of the spiralic pattern – as betrayed by the left side of the box of #42, where the virtual hexagonal symmetry of the pattern is allowed to unfold.

The buttons of #43 carry two different patterns, both of which combine quadrangular symmetry and circles in a sophisticated way. On one set (buttons labeled 685) a pattern of concentric circles is transformed into a kind of meander, symmetrical about two mutually perpendicular axes and

[19] The idea that circles have a hidden affinity to hexagonal symmetry is difficult to get at unless one makes experiments with a compass with constant opening. We may hence guess that the pattern of #40–41 presupposes inspiration from something like #38 but made in a different medium – possibly rope constructions of regular hexagons or related figures (or familiarity with regular tree-planting, for that matter).

*Photograph # 41. ArchM, Museum N°261. Mycenae, grave circle A, shaft grave 4, 1580–1550.*

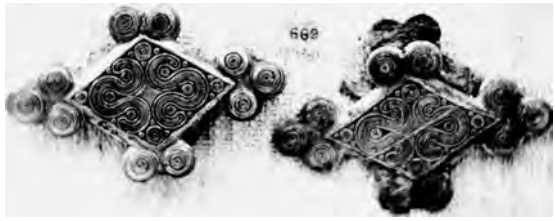
*Photograph # 42. ArchM, Museum N°808. Mycenae, grave circle A, shaft grave 5, 1580–1550.*

*Photograph # 43. ArchM, Museum N°682+685. Mycenae, grave circle A, shaft grave 5, 1580–1550.*

*Photograph # 44. ArchM, Museum N°334. Mycenae, grave circle A, shaft grave 4, 1580–1550.*

unchanged when an inversion (in naive formulation, an "inward-outward-reflection") is followed by a rotation of 90°. On the other set (labeled 682) a square is filled out by a combination of smaller and larger circles (the

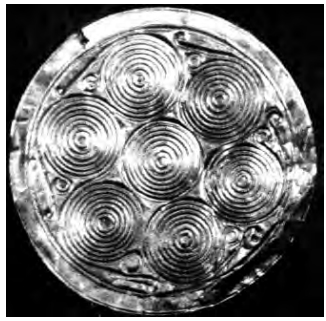
latter combined two by two into incomplete "figures of eight"). Due to the skilful combination the completion of the square becomes mathematically coherent, even though the inscription of the square in an outer circle remains eclectic. The buttons in #44 exhibit hexagonal symmetry, but possess the same invariance under inversion + rotation as the first type in #43 (here under a rotation of 30°, not 45°).




*Photograph # 45. ArchM, Museum N°669. Mycenae, grave circle A, shaft grave 5, 1580–1550.*

The two pieces of #45 show a three-circle variant of the "incomplete figure of eight", adapted to inscription in a double triangle (the lower piece is indeed composed of two very precise equilateral triangles). Probably for reasons of material stability and aesthetic harmony, the diameters of the circles which are added externally deviate from what could be expected if triangular symmetry had been the sole and overriding concern. The deviations seem to be mainly *a posteriori*, however, and not *a priori* as in the third millennium items: high mathematical symmetry is the starting point.

The same relative but not absolute primacy of the mathematical structure over non-mathematical aesthetic considerations is seen in a number of gold roundels from Shaft Grave III (belonging to the "second" generation). With #38 (our starting point), #46, #47, #48, #49, #50 and #51 form a continuum in this respect. #38 was purely "mathematical", being built on what could be extended to a geometrical ordering of the complete plane by means of intersecting circles. The systems of concentric circles of #46 are arranged according to the same hexagonal symmetry, but the outer circles of the six systems in the periphery are opened in order to make the whole configuration fit harmonically within the circular border. #47 presents us with another solution to this problem, reminding



*Photograph # 46 (left). ArchM, Museum N°10. Mycenae, grave circle A, shaft grave 3, 1550–1500.*
*Photograph # 47 (right). ArchM, Museum N°14. Mycenae, grave circle A, shaft grave 3, 1550–1500.*

*Photograph # 48 (left). ArchM, Museum N°18. Mycenae, grave circle A, shaft grave 3, 1550–1500.*
*Photograph # 49 (right). ArchM, Museum N°18. Mycenae, grave circle A, shaft grave 3, 1550–1500.*

of the triple circles of #45. In #47, however, the deviation from the basic mathematical pattern becomes more important than in #46: there is no longer any simple relation between the diameter of the concentric systems and the diameter of the circle which surrounds them, while those of #46 have a ratio of 1:3, and while the diameter of the small circles of #38 equalled the width of the "petals".

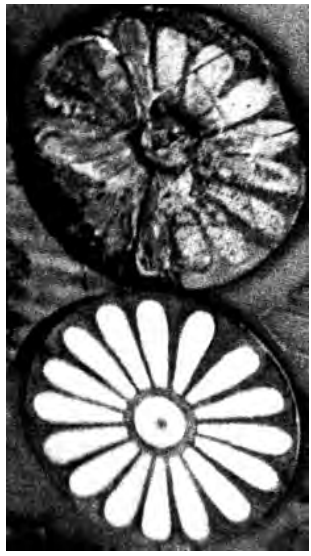
#48 conserves the hexagonal rotation symmetry and remains abstract, but mirror symmetry has been given up, and the six figures forming the pattern have no simple mathematical description. With #49 we enter

the realm of figurative decoration – yet the octopus is more symmetric and regular than any octopus seen before or after, and indeed almost as symmetrical as at all permitted by the motif. Firstly, the axis of the body is a perfect symmetry axis; secondly, the spiraling arms are arranged in an almost regular octagon, and the centre of the outer circle and of this octagon coincides as precisely as can be measured with the foremost point of the body.

*Photograph # 50 (left). ArchM, Museum N°4. Mycenae, grave circle A, shaft grave 3, 1550–1500.*
*Photograph # 51 (right). ArchM, Museum N°13. Mycenae, grave circle A, shaft grave 3, 1550–1500.*

The bee of #50 exhibits only a simple mirror symmetry, which is all the motif allows – yet closer inspection of the figure reveals unexpected hidden mathematical regularities (quite the reverse of the "geometrical impressionism" of the third millennium); in this sense, the motif only serves as a pretext. Once more, the centre coincides with the foremost point of the abdomen (the only visible part of the body). The abdomen, furthermore, is fitted into the right angle formed by the hind edges of the wings; the front edges of these are curved but approach the prolonged hind edges asymptotically, and the whole configuration is thus determined by a pair of mutually perpendicular axes through the centre of the circle; finally, the insect is provided with ten wings in order to make all this possible. Only the stylized leaf on #51 accepts the requirements of the motif and relinquishes central symmetry completely.

The symmetry of the octopus and the ten-winged bee is not only found on the gold roundels. In #52 we see a rosette with 16 petals (most clearly



*Photograph # 52. ArchM, Museum N°556. Mycenae, grave circle A, shaft grave 4, 1580–1550.*
*Photograph # 53. ArchM, Museum N°564+557+562. Mycenae, grave circle A, shaft grave 4, 1580–1550.*

to be seen on the reconstruction below the original), and in #53 a piece of a greave decorated with mutually perpendicular sets of parallel lines. As in the case of the bee, these are drawn at an inclination of 45° from the "vertical" line of the greave, and unlike seemingly related patterns from the fifth through the third millennium they are drawn with exactly four lines in each set, and with the distance between the sets equal to the width of each set. The use of precise geometrical relationships was obviously no prerogative of goldsmiths and jewellers.

Goldsmiths, however, have provided us with the most astonishing examples of geometrical attention. #54 is a sword blade decorated with the same pattern as #42, but under particular geometrical conditions. In contrast with many of the eclectic pieces from earlier periods, the pattern is adapted coherently without losing its own character (thus in a generalized sense, *conformally*) to these conditions which come to function as genuine *boundary conditions* not only in the direct but also in the mathematical sense. In #55, the tip of another sword blade, a

series consisting of three lions is subjected to the same treatment, showing that even this series is basically dealt with by the artist as a geometrical pattern, irrespective of the naturalistic appearance of the single lion.

*Photograph # 54. ArchM, Museum N°744. Mycenae, grave circle A, shaft grave 5, 1580–1550.*

The final specimen from sequence I is a fragment of a thirteenth-century fresco from Mycenae. The picture (#56) does not show it clearly, but the original in colour demonstrates that the Mycenaean artists would have some kind of abstract notion (whether explicit or not we cannot know[20]) of the invariance under an inversion+rotation discussed in connection with #43–44. Both the necklace and the bracelets of the woman consist of red and yellow pearls; in both, the pearls are ordered in groups of three. Yet while the colour sequence of the necklace is $\cdots - y \cdot r \cdot y - y \cdot r \cdot y - \cdots$, that of the bracelets is $\cdots - r \cdot y \cdot r - r \cdot y \cdot r - \cdots$. In this case, switching both the two colours and the two positions thus leaves the system unaltered. The mathematical coherence of the geometrical decorations of #37-55 thus appears to reflect a more general mode of thought.

Sequence II, #57–70, shows that the case of Mycenaean ceramic decorations was different.

The sixteenth-century vase in #57 is very close to the style of the various pieces from Melos from the same or slightly earlier periods; the style is imported, if not the vase itself, and almost as different as can be imagined from that of sequence I.

[20] Exactly the same commutative group can be dug out from Adalbert von Chamisso's "Canon" [*Werke* I, p. 85]:

*Das ist die Not der Schweren Zeit!*
*Das ist die schwere Zeit der Not!*
*Das ist die schwere Not der Zeit!*
*Das ist die Zeit der schweren Not!*

– but nobody would suspect Chamisso of having thought of this elegant game as a piece of mathematics.

*Photograph # 55. ArchM, Museum N°395. Mycenae, grave circle A, shaft grave 5, 1580–1550.*

*Photograph # 56. ArchM, Museum N°11671. Fresco, Mycenae, 13th c.*

*Photograph # 57. ArchM, Museum N°199. Mycenae, grave circle A, shaft grave 1, 1550–1500.*

The further development of Mycenaean ceramic decoration presents us with an increasing interaction with the geometrically regular tradition. Already the fifteenth century palms and ivys of #58 have lost some of the free movement of former times, organized as they are within an

*Photograph # 58. ArchM, Museum N°7107. Mycenaean ware, 'palace style', 15th c.*

approximate mirror symmetry; each of the two birds of #59 are still very soft in their lines, but once again the composition as a whole is tendentially symmetric. Much more constrained by symmetry are the octopus figures of #60 and #61, the contrast of which to the octopus of #34 is about as great as can be. Beyond the symmetry of the animals we also notice the emergence of non-figurative elements: hatchings and zigzag-lined ribbons, as well as spirals and systems of concentric circles used as eyes.

At closer inspection, however, these elements of "geometric" decoration turn out to be widely removed from the mathematical geometry of the sixteenth-century items discussed above. The number of lines in the bands of parallel lines between the arms of the octopus in #61 varies, it seems, according to nothing but the aesthetic sensibility of the artist (certainly a most pertinent criterion in an object which must somehow have been meant to be beautiful, and a better choice than obsession with arithmetical uniformity); the spiraling eyes of #60 do not follow the symmetry of the figure – both approach the centre in a clockwise movement.[21] The "mathematical" experiments of the sixteenth-century Mycenaean court appear to have been left behind; once more the geometrical regularity

[21] It is true that the two fish below the octopus exhibit the same translational symmetry; in contrast to what could be argued in the case of #10, it is therefore not to be excluded that this piece presents us with an intentional clash between two irreconcilable symmetries.

However, two similarly oriented fish can be found in the kindred #61, in which the pattern of hatchings between the arms exhibits no similar translation symmetry; intentional symmetry breaking in #60 therefore remains an unconvincing possibility.

*Photograph # 59. ArchM, Museum N°1275. Mycenae, Acropolis, 14th–13th c.*

has become one of immediate visual impression (but no hidden governing principle, *cf.* note 17). It is tempting to see this stylistic simplification as a symptom of the decline of courtly wealth and of the disintegration of the Mycenaean social system. Some of the elements (*e.g.*, the particular constitution of the systems of concentric circles) are so close to Early Cycladic specimens that we must presume a survival of these forms outside the courtly workshops; such survival is even more obvious in #62, as impressionistic as anything similar from the third millennnium, and clearly akin to #23.

"Decline" is also visible in the drawings of humans and animals in #63–65, if one compares them with the pictorial representations of hunting and war scenes of the sixteenth century (one example can be discerned on #42; a better reproduction is [Marinatos 1976: Plate 220]) – and the continuation of certain stylistic features indicates that comparisons can legitimately be made.[22] Inside the drawings of living creatures, many of

[22] Thus, the Donald-Duck faces and the thighs of the Tiryns-vase (#64) are both developments of less abnormal characteristics of the hunters portrayed on the blade of a sixteenth-century dagger (Museum number 394; photo in [Petrakos 1981, p. 30]). Other items with figurative decorations from the early period are so close to Near

*Photograph # 60 (left). ArchM, Museum N°6193. Attica, Perati, 13th c.*
*Photograph # 61 (right). ArchM, Museum N°9151. Attica, Perati, 13th c.*

*Photograph # 62. ArchM, Museum N°3559. Mycenae, chamber tomb E, 15th–13th c.*
*Photograph # 63. ArchM, various museum numbers. Mycenae, Acropolis, fragments, 'pictorial style', 13th–12th c.*

the fragments of #63 show hatchings, chessboard patterns and other features reminding of the Neolithic decorations discussed above. The same holds for the Tiryns vase (#64), which at the same time shows hints of that repetitiveness which has become the dominant characteristic of the "vase of warriors" (#65) and of the non-figurative #66 (the repeated element

Eastern styles, it is true, that they must be considered as borrowings and to be thus less relevant to a diachronic comparison.

of which can be compared with the constituents of #46). Since repetitiveness is in itself an elementary but visually obvious variant of geometrical regularity, the whole tendency of Late Mycenaean figurative decoration can be seen as a gradual sliding-back into primitive geometry – as could perhaps be expected in a situation where the social division of labour itself became less complex (*cf.* note 12), but which does not preclude that certain pieces are very finely made and very beautiful – as is #67.

*Photograph # 64 (left). ArchM, Museum N°1511+10549. Tiryns, Acropolis, 13th c.*
*Photograph # 65 (right). ArchM, Museum N°1426. Mycenae, Acropolis, mid 12th c.*

*Photograph # 66 (left). ArchM, Museum N°12163. Mycenae, Acropolis, 14th–13th c.*
*Photograph # 67 (right). ArchM, Museum N°7626. Mycenae, "house of the oil merchant", 13th c.*

## FROM 'GEOMETRIC STYLE' TOWARD, AND AWAY FROM, GEOMETRY

The "Geometric" style that develops from the Submycenaean age

(twelfth to earlier eleventh century) onwards is thus, in a way, the ultimate consequence of developments which had started in the outgoing Mycenaean era; #66–70 demonstrate this clearly. The extreme geometrization of pieces like #75–77 is, so to speak, the point toward which the Late Mycenaean stylistic changes (as continued during the Submycenaean and Protogeometric phases, #71–74) are directed if they should end up in a fully coherent style; that they should end up so was of course no historical necessity, and eventually it turned out to be only an ephemeral phenomenon.

*Photograph # 68 (left). ArchM, Museum N°3493. Mycenae, chamber tomb, 15th–13th c.*
*Photograph # 69. ArchM, Museum N°5197. Attica, end of 13th c.*

The Submycenaean and Protogeometric phases are represented by a few photos only, but some essential points can be described in words.[23] The broad black bands so characteristic of Early Geometric pottery go back to "Mycenaean IIIB" (c. 1300 BCE) – some characteristic specimens are depicted in [Finley 1970, p. 64]. The systems of concentric circles, too, have Mycenaean antecedents, as demonstrated by #70. They are immensely popular on Protogeometric pottery (c. 1050 to c. 900), but they are also found on earlier eleventh and twelfth-century ware. In the tenth

[23] The description is based on the exhibitions of Attic ware in the Oberländer and Agora Museums in Athens. Some of the plates in [Whitley 1991] allow similar observations.

century they are, as a rule, drawn by means of a multiple-brush compass, as on the Mycenaean #70. During the twelfth and most of the eleventh century, on the other hand, this "professional" technique is absent, and the circles are drawn by hand. Throughout the period the centres are often located at the edge of a black band, and only semicircles are drawn. Inside the inner circle or semicircle, a straight-line figure (an "hour-glass" or a cross) is often found.

*Photograph # 70 (left). ArchM, Museum N°3198. Mycenae, chamber tomb, 15th–13th c.*
*Photograph # 71 (right). ObM, Athens, Submycenaean, Grave N°136, 11th c.*

Many constituents of the Early Geometric style – zigzag-lines, meanders, hatchings, etc. – are evidently comparable to elements of earlier styles. Their reappearance after their virtual absence during the Submycenaean and early Protogeometric periods looks like a consequence of an inner logic of the style and/or its cultural context. At the same time, several of these constituents are so specific that we must suppose them to represent a surviving tradition – thus the hexagonal rosette of #38 and #78, or the eight-fold "flower" at the top of #80, which is already present in this precise form on a fourteenth-century gold roundel in the Agora Museum. Revivals of waning or rarefied traditions, however, only come about if these become adequate once again within a changing technological, social or cultural horizon.

Even the incomplete repetitiveness of abstract as well as figurative decorations comes so close to the idea of certain late Mycenaean items that

*Photograph # 72. ObM, Athens, Early Protogeometric, grave N° hs 130, 11th c.*

we may guess at the existence of a continuous undercurrent which, under new conditions, rose to prominence once again (compare #64–65 with #78 and #79; sixteenth-century examples are found, *e.g.*, in [Marinatos 1976, Plates LII and p. 218]).

The sixteenth-century Mycenaean court style and the geometric splendour of the eighth century *may* thus be sophisticated manifestations, in different cultural and social contexts, of certain basic traditions, practices or ideas. Yet in spite of this possibly shared background the two are conspicuously different. Whereas the Mycenaean court style had pursued mathematical regularity, regularity below the level of the visually obvious is a minor concern in Geometric pottery. Attentive scrutiny of the repeated elements of a repetitive pattern makes their apparent identity fall apart: the number of strokes in a hatching or a herring-bone pattern varies; one vertical zigzag-line begins in the upper left corner, one to the right, and one in middle; one has 21 apices, another 24; etc. The underlying geometry is different from that of the outgoing third millennium, it is true; the emphasis on visual impression rather than precisely controlled regularity or deliberate breach of symmetry, however, is the same. *If* the Mycenaean court style and the Geometric style are manifestation of a common background vision or aesthetics, then the Mycenaean inclination toward genuine mathematization seems not to belong to this shared background, at least not as it survived in the early first millennium.

As at the turn of the third millennium, the further development of the

*Photograph # 73. ArchM, Athens, Museum N°18076. Late protogeometric, undated in the Museum.*

*Photograph # 74. ObM, Athens, Transition Protogeometric-geometric, cremation burial of a warrior, 875–850.*

Geometric style was to change geometric into more living forms. Already in the eighth century certain rosettes consist of indubitable leaves, and a culmination of this trend is seen in #80 (seventh century). Still, the "life" of this amphora is characterized by being a transformation not of spirals (as the second millennium aquatic plants etc.) but of a stiff linear geometry. With or without influence from the "orientalizing" style (see #81), however, further development through repetitive (#82) and gradually less repetitive (#83) human and animal forms was to lead to the free artistic form of Classical vase painting, in which the geometrical

*Photograph # 75. ArchM, Museum N°216. Attica? 850–800.*

*Photograph # 76. ArchM, Museum N°185. Middle Geometric II, 800–760.*

*Photograph # 77. ArchM, Museum N°812. Dipylon, Athens, Late Geometric I A, 760–750.*

*Photograph # 78. ArchM, no museum number indicated. Dipylon, Athens, Late Geometric 1B, 750–735.*

*Photograph # 79. ArchM, Museum N°17935. Attica? 720–700.*

knowledge of the artist is only used silently for balancing the picture and making it dynamic (if we disregard the meanders which occasionally border the figurative paintings).

A corresponding development is seen in the sculptural arts. The *kouroi* of the early Archaic age appear to be strongly inspired by a late variant of the Egyptian canonical system. Not only are the numerical proportions between the parts of the human body observed, but the body's whole posture is determined so as to correspond to a specific square grid (see [Iversen 1971, p. 75–77]; compare in particular #84 with the *kouros*

*Photograph # 80. ArchM, Museum N°77. (19.762). Attica? Archaic period, 700–600.*

*Photograph # 81. ArchM, Museum N°12130 and 12077. Eretria, 'Orientalizing style', 7th c.*

*Photograph # 82. ArchM, Museum N°530. Attica, 600–550.*

*Photograph # 83. ArchM, no museum number indicated. Pharsala, Thessaly, c. 530.*

*Photograph # 84. ArchM, Museum N°3645. Cape Sounion, Attica, 600–590.*

inscribed in a square grid on p. 77). At this moment, mathematical regularity (here primarily proportion) is thus not only a governing principle but also a visually outstanding feature. Sculptures from later periods are still made according to those proportions which were deemed harmonious and therefore beautiful. Yet the system became less fixed; being now a subservient means to achieve the artistic end, geometry became

*Photograph # 85. ArchM, Museum N°15101. Attic bronze, c. 460.*

an underlying regulative force. Sculptures like #85 and #87 demonstrate to which extent the posture of the human body was made an expression of character and emotion, freed from all *visible* mathematical constraint. Even when Nature was stylized into abstract pure form (as in #86, from the Poseidon temple in Sounion) the shapes which occur are quite different from those simple curves – the circle and the straight line – which were canonized in theoretical geometry precisely during the epoch when the Sounion temple was built. At the time when mathematics evolved into an autonomous intellectual pursuit, and when Oenopides and Hippocrates started the development which was to end up as axiomatization, the artists for their part stepped into a realm of forms far beyond the reach of scientific geometry. At least one reason for this emerges from Vitruvius' discussion of the dimensioning of columns (*De architectura* III.iii.10 – ed., trans. [Granger 1931, pp. I, 176–181]): these have to be narrower at top than at bottom in proportions depending on their height; they have to swell in the middle; those in the corners have to be a bit thicker than the rest – and all because "what the eye cheats us of, must be made up by calculation". This purpose was served much better by concrete rules

*Photograph # 86. ArchM, Museum N°1112. Poseidon Temple of Sounion, c. 440.*

based on experience and reduced to elementary numerical formulas than by any geometrical theory. Which mathematical theory would ever be able to tell the artist that the line defined by the three heads in #87 should descend toward the right (as it does indeed) in order to confer the feeling of calmed passivity involved in deep sorrow, while descent toward the left could have had quite inappropriate implications?

## A MORAL?

If we are to learn any lesson from our story, a bird's-eye view of the development may be useful. The Old European Middle Neolithic confronts us with simple patterns: zigzag lines, rhombs, etc. No effort is made to achieve geometrical coherence between the various parts of a decoration. Further on greater fantasy manifests itself in the choice of forms, and various symmetries and other invariants are explored. From a start in

*Photograph # 87. ArchM, Museum N°723. Athens, early 4th c.*

pure decoration the geometrical pattern develops toward structural experiments.[24] This development, however, is never carried to its mathematical consequence: eclectic decoration endures, the style remains one of geometrical impressionism. With the partial exception of #17, no attempt is ever made to explore the inherent formal ("mathematical") properties of the shapes and symmetries dealt with. Throughout the period decorative and artistic concerns are overriding (together probably with symbolic and similar concerns which, however, could equally well express themselves one way or the other).

As the Middle Cycladic offspring of Old Europe falls under the influence of Minoan Crete (itself largely an Old European offspring), this dominance of artistic concerns undergoes a qualitative leap: instead of introducing

[24] It may be worthwhile repeating once again that this distinction only concerns one axis in the multi-dimensional grid in which the character of the decorations can be located; in particular it does not anticipate the answer to questions concerning their possible symbolical function.

figurative painting as a supplement to the old geometrical decoration, the geometrical design itself is changed and vitalized. The pattern is transformed into naturalistic or quasi-naturalistic art; what remains of geometrical principles is mainly the use of deliberately broken symmetries that serve to balance the composition while keeping it tense.

The "native" Mycenaean tradition is different. We first met with it in the shaft-graves of Mycenae where, over one or two generations, a high level of regularity developed into genuine mathematical structuring. Later Mycenaean art becomes less mathematical, as we see it in the ideals which are pursued in the "normalization" of the borrowed Minoan vase painting style; the development through Protogeometric and Geometric art suggests, however, that the early Mycenaean bloom is a high-level manifestation of a general cultural substrate where straight lines, circles, and quadratic, hexagonal and octagonal (and even abstract) symmetries are important. Even though the geometrical impressionism of the Geometric period never evolves into structural mathematical inquiry (but eventually, like the impressionism of the third millennium, into "art"), this second bloom of professional art among the Greek-speaking tribes shares some fundamental characteristics with its Mycenaean predecessor. For some reason Greek culture maintained an interest in circles, squares, hexagons and octagons for more than a thousand years *before* theoretical geometry emerged.

That geometry was one of the fields that were made the objects of *theory*, along with more obvious fields like cosmology and health, may perhaps owe something to the existence of such a substrate. The name given to the subject, it is true, demonstrates that the "metric" component of geometrical thought was assumed by the Greeks themselves to be its essence. As suggested by Wilbur Knorr [1975, pp. 6f and *passim*] and others, however, the strand leading to *Elements* II etc. did not constitute the whole rope, and *Elements* III and its kin could be the ultimate outcome of theoretical reflection inspired by favourite shapes – just as the "metric" component may be the outcome of theoretical elaboration of Near Eastern mensuration geometry (this is not the place to investigate the interaction and mutual fecundation of the two currents).

Even so, although this kind of inspiration is indisputably possible, there is no path leading from the decorations of Geometric vases to theoretical

geometry. This is evident already from plain chronology, since Geometric vases disappear long before anybody imagines theoretical geometry to have arisen; moreover, it is hardly possible to find any element in mature Geometric art which points to the specific interests of Greek (metric or non-metric) geometry. Geometric art reflects interest in geometrical shapes and symmetries, but in contrast to Mycenaean art it is not a medium through which these are submitted to further formal (and hence "mathematical") scrutiny or experiment.

Similar conclusions can be drawn regarding any geometrical impressionism. Geometrical impressionism demonstrates the presence of an aesthetics of visual order and (generalized) symmetry, but it also proves that the artist is satisfied by fulfilling the requirements of this aesthetics, and is not interested in further investigation of formal properties.

Decorative patterns are not always impressionistic, and the decorations of many cultures not discussed here can be regarded in full right as expressions of formal investigation and experiment.[25] The moral of the

[25] Indeed, much of the decorative art of Subsaharan Africa contains such formal investigation and experiment. This has been amply demonstrated by Paulus Gerdes and his collaborators in a number of books, as I have pointed out in reviews, which I quote here:

"All the examples explored by Gerdes (and sub-Saharan geometrical decoration in broad average as far as the reviewer is aware) belong to the [...] type" which "bears witness of deliberate explorations of symmetries and other formalizeable properties of figures; its actual drawings need not be very precise, but they contain an underlying formal structure" ([Høyrup 1996], review of [Gerdes 1994]).

"The [...] weavers" of "sipatsi: handbags woven from white and coloured straw exhibiting geometrical strip patterns" are "very conscious of the numerical principles underlying the patterns and very critical of irregular patterns arising from sloppy counting or insufficient mental calculation. Mathematical regularity is thus anything but a mere result of the constraint inherent in the technique" ([Høyrup 1997a], review of [Gerdes & Bulafu 1994]).

"[...] the specialists in question do not look at themselves as 'mathematicians', a role for which traditional society has no space; but many of the patterns shown in the book exhibit symmetries that bear witness of intense reflection on formal properties of patterns. These are not restricted to invariance under the combination of reflections in lines and points, translations, and rotations, but also involve abstract invariances under combinations of spatial transformations and colour inversion (or even switches between monochrome and hatched) and symmetry breakings that arise when locally symmetric configurations are inserted in a global pattern with a different symmetry." ([Høyrup 1997b, review of [Gerdes 1996]).

The *sona*, line drawings made in the sand, "represent specific objects, situations, proverbial sayings, or even stories, and they were an essential part of the teaching surrounding the adolescential circumcision. All adults would therefore be familiar with

present tale is, firstly, that we should be careful not to extrapolate from every piece of geometrical decoration to such extensive symmetries which may be superimposed on its pattern but which are not needed to explain it (and, worse, may be contradicted by its details); secondly, that no necessity leads from an *aesthetics of forms to formal investigation of forms, nor from formal investigation of forms to integration with mensurational geometry* or *into mathematics as a broader endeavour* (whether provided with proofs or not).

We should respect that not everybody prefers the ideals expressed in #37 (even if we disregard the warrior on his chariot) to those inherent in #34.

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some of the simpler patterns, but the more complex ones would only be known by a restricted group of specialists who kept them jealously as secrets. In consequence, the tradition is almost lost today, and the first part of the book therefore aims both at presenting and analyzing a large number of sona documented in the literature, and at reconstructing the algorithms and composition principles which allowed the masters to perform them (as required) without the least hesitation" ([Høyrup 1999], review of [Gerdes 1997]).

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